EQUATIONS OF THE ELECTROMAGNETIC FIELD IN DISPERSIVE MEDIA (as applied to Transient Geo-Electromagnetics)

Felix M. Kamenetsky

Ludwig-Maximilian-University of Munich, Germany. Department of the Earth and Environmental Sciencies. Section Geophysics.
*E-mail: **kamen@geophysik.uni-muenchen.de**.*

Summary

The frequency/time dispersion of the electric properties of rocks is intensively investigated in geo-electromagnetics. In particular, the strong disperson effects in transient geo-electromagnetics, like non-monotonous transient response, sign reversals etc., have been considered by many authors. The aim of this paper is to describe the electromagnetic field in dispersive media in the most general form. The validility of replacing constant parameters with frequency/time variable ones is the subject to analysis as well. It is emphasized first of all that Maxwell's equations written in canonical form are universal ones to describe the electromagnetic (EM) field both in non-dispersive and dispersive media, provided material correlations are presented by Duhamel integrals. It is shown that in the time domain additional conduction and displacement currents appear due to changes of conductivity and permittivity parameters with time. Methods of changing constant parameters with time variable ones are principally not correct in time domain and correct in frequency domain. It is shown, however, that separate analysis of dispersion of conductivity or permittivity is impossible. EM field defined by the complex admittivity including both frequency dependent conductivity and permittivity is to be analyzed.

1. Introduction

The object to analysis by geo-electromagnetics is the response due to a geo-electrical medium. It is normally a half-space consisting of homogeneous entities. To simplify the analyses we believe that the rock's electromagnetic properties inside the entities possess no space-gradient and no anisotropism. Rocks are considered here to be macro-homogeneous, a result of being natural composite micro-heterogeneous (multi-component and/or multi-phase) media. They are characterized by effective values of electromagnetic parameters (conductivity σ, permittivity ε and magnetic permeability μ) obtained by averaging the electromagnetic (EM) field in a relatively small volume, much larger, however, then the dimensions of separate components composing the medium. It was determined, both experimentally and theoretically, that the media of the sort display the dispersion of the parameters above, which are in this case time/frequency dependent in time/frequency domains respectively. This dispersion is intensively investigated in geo-electromagnetics.

The low frequency dispersion of conductivity investigated in quasi-stationary time domain, where common displacement currents are neglected, is known as induced polarization (IP). In particular, the numerous strong IP effects in transient geo-electromagnetics, like non-monotonous behavior of transient response, sign reversals etc. obtained by laboratory and field measurements, as well as by mathematical modeling, have been reported by many authors. Typical results of physical studies of EM transients inductively energized in rock samples and affected by IP (Kamenetsky and Novikov, 1997) are given in Figure 1. Sign reversals are clearly seen at different time positions for different rock's types with one and the same ohmic resistivity 3.3 Ohm·m.

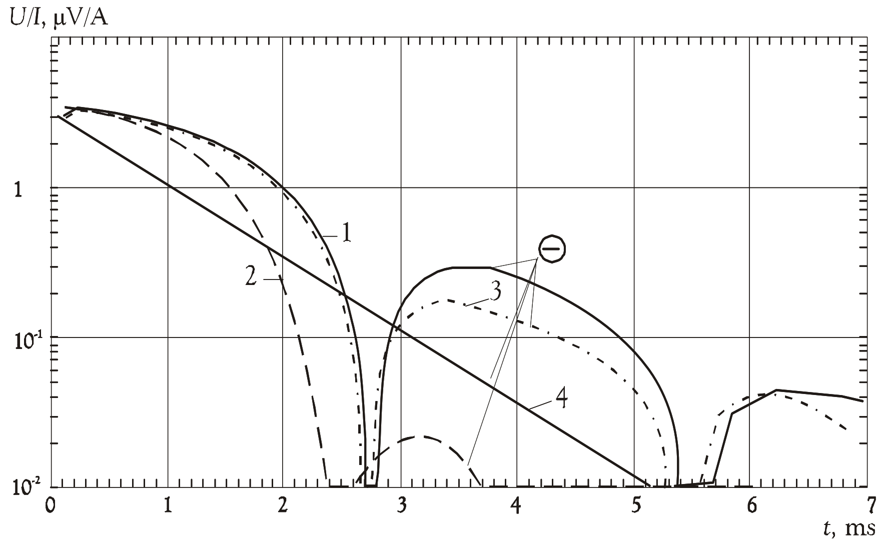

Fig. 1. Induction transients of samples with one and the same resistivity:
1 – clay, 2 – sand, 3 – clay-sand, 4 – active equivalent (3.3 Ohm·m).
Θ- negative response.

Additional studies have been done to examine the phenomenon of the unusually high resolution of transient geo-electromagnetics (sometimes comparable to that of seismics) experimentally observed while exploring sedimentary formations mainly for oil and gas (Safonov et al., 1996). It was shown by mathematical modeling that the IP effect can be considered as one of the probable causes of the phenomenon. However, its mechanism was not explained in the framework of classic electrodynamic theory and is still not fully understood. Terms like "non-classical geo-electromagnetics/geo-electrics" have been also introduced to emphasize that the mentioned above polarization phenomena cannot be described by Maxwell's equations (Svetov, 1995).

As for IP effects in the low frequency EM, it was much less investigated and reported, especially in studying sedimentary formations. For example, it is almost always ignored in magnetotellurics (MT), except for very few works like (Porstendorfer, 1987). For sedimentary rocks the so-called "fast IP" are typical with a very short time constant (mach shorter then a period of the MT variations). That means that the sedimentary rocks are completely polarized at such low frequencies and characterized by the stationary value of conductivity $\sigma_0 = \sigma_\infty(1-m)$, where m is chargeability and σ_∞ is the true (with no polarization) conductivity at higher frequencies. The stationary conductivity is, therefore, in most cases the only parameter, which is recovered by MT and other low frequency EM methods. The possibility to extract in this case additional information about IP parameters is a separate subject of research which is outside the scope of the present paper.

The study of IP effects in geo-electromagnetics includes: (1) experimental investigations of rock's polarization properties, (2) investigation of the same on the basis of models where the nature of dispersion is known, and (3) theoretical analysis (as well as mathematical modeling) of an affection of the EM data by IP. The combination of (2) and (3) was developed in the fundamental work of Sheinmann (1969) by introducing the ion's diffusion currents in the fluid phase of rocks into the second Maxwell equation. This approach introduces numerous new parameters for each particular type of polarization. The phenomenological approach is most commonly used (Kormil'tsev, 1989, Wait, 1959), where different IP effects are described by a small number of phenomenological parameters, such as chargeability and time constant, irrespective of the polarization nature.

To solve electrodynamic problems for dispersive media in the time domain, Maxwell's equations are normally transformed into equations of the second order (Telegrapher's, or heat-flow equation in the quasi-stationary case), first with parameters σ and/or ε independent of time. Afterwards, the change of the EM field with time change of parameters is taken into account by different ways of replacing them with time dependent ones. Otherwise, the time domain solutions are obtained by the spectral method, i. e. by solving Helmholtz equation first with frequency independent parameters σ, ε, and then by replacing them with frequency dependent ones and applying the inverse Fourier/Laplace transform to the solution in frequency domain.

The aim of this paper is to describe the electromagnetic field in dispersive media in the most general form. The validity of replacing constant parameters with variable ones is the subject to analysis as well. Firstly, we pay attention of the readership to that Maxwell's equations themselves written in canonical form are not limited to non-dispersive media and valid in dispersive media as well. As for the equations of the second order, we show that the replacing approach above is valid in frequency domain and principally not valid in time domain. These equations for dispersive and non-dispersive media are found to be different in time domain and the same in frequency domain. We prove the same by practical example of the dispersion with a simple known nature.

2. Maxwell's equations

The canonical form of the system of Maxwell's equations in medium with no free electric charges is the following:

$$\mathrm{I.}\ curl\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},\ \mathrm{II.}\ curl\mathbf{H} = \mathrm{j_c} + \frac{\partial \mathbf{D}}{\partial t},\ \mathrm{III.}\ div\mathbf{D} = 0,\ \mathrm{IV.}\ div\mathbf{B} = 0\ . \quad (1)$$

Here and below:

E, **H** are electric and magnetic fields,

D, **B** – electric and magnetic inductions,

$\mathbf{j_c}$ – the conduction current density.

Assuming the medium parameters σ, ε, μ are non-dispersive ones, i. e. independent of time, then the system (1) is accompanied by equations of material correlations of the form:

$$\mathbf{j_c} = \sigma\mathbf{E}, \quad \mathbf{D} = \varepsilon\mathbf{E}, \quad \mathbf{B} = \mu\mathbf{H}\,. \tag{2}$$

This independence, however, is not inherent into the system (1) of equations I-IV above which are valid for dispersive media as well, irrespective of the nature of dispersion. As for material correlations (2), they are to be generalized.

IP was firstly introduced into Maxwell's equations by Kormiltsev (1989). Following Kormiltsev, the system (1) is to be accompanied in this case by equations of material correlations presented in form of Duhamel integrals for combinations of EM field components with the so-called "after-effect" functions.

To develop this idea we prefer more compact equivalent form of the same using the Duhamel integrals for combinations of electric field component directly with parameters σ and ε (Kamenetsky. 2002). This approach is, to our mind, not only more natural, but will afford a better understanding of the phenomena in question. We shall use the symbol * for the Duhamel integral, that means for the operator

$$f * \mathbf{F} = \frac{d}{dt}\int_0^t f(\lambda)\cdot \mathbf{F}(t-\lambda)d\lambda\,, \tag{3}$$

where F and *f* are field component and parameter of the medium correspondingly. We shall use also the following auxiliary formulae while differentiating Duhamel integrals (see Appendix A):

$$f * \mathrm{F} = \mathrm{f}(0)\mathrm{F}(t) + \int_0^t \mathrm{F}(t-\lambda)\frac{\partial f(\lambda)}{\partial\lambda}d\lambda\,, \tag{A1}$$

$$\frac{\partial}{\partial t}\left(f * \mathrm{F}\right) = f(0)\frac{\partial \mathrm{F}(t)}{\partial t} + \frac{\partial f(t)}{\partial t}\Big|_{t=0}\mathrm{F}(t) + \int_0^t \frac{\partial^2 f(\lambda)}{\partial\lambda^2}\mathrm{F}(t-\lambda)d\lambda\,, \tag{A2}$$

$$\frac{\partial}{\partial t}\left(f * \mathbf{F}\right) = f * \frac{\partial \mathbf{F}}{\partial t}\,. \tag{A3}$$

In the general case (including both dispersive and non-dispersive media) instead of (2) one has to write

$$\mathbf{j_c} = \sigma * \mathbf{E}, \quad \mathbf{D} = \varepsilon * \mathbf{E}, \quad \mathbf{B} = \mu\mathbf{H}\,.^{1} \tag{4}$$

By taking into account the correlations (4) the system (1) of Maxwell's equations can be rewritten for dispersive media in the following generalized form

---

[1] We confine ourselves here with the time/frequency independent magnetic permeability.

$$\text{I.}\, curl\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},\ \text{II.}\, curl\mathbf{H} = \sigma * \mathbf{E} + \frac{\partial(\varepsilon * \mathbf{E})}{\partial t},\ \text{III.}\, div(\varepsilon * \mathbf{E}) = 0,\ \text{IV.}\, div\mathbf{B} = 0\,. \quad (5)$$

Since ε is not space-dependent parameter, $div(\varepsilon * \mathbf{E}) = grad\varepsilon * \mathbf{E} + \varepsilon * div\mathbf{E} = \varepsilon * div\mathbf{E}$, and the equation III in (5) can be replaced by $div\mathbf{E} = 0$. Then finally

$$\text{I.}\, curl\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},\ \text{II.}\, curl\mathbf{H} = \sigma * \mathbf{E} + \frac{\partial(\varepsilon * \mathbf{E})}{\partial t},\ \text{III.}\, div\mathbf{E} = 0,\ \text{IV.}\, div\mathbf{B} = 0\,. \quad 6)$$

There are, therefore, certain features which are to be taken into account while solving electrodynamic problems in dispersive media. As for non-dispersive media, it can be easy shown, that material correlations (4) are reduced to (2) assuming parameters *σ*, *ε* independent of time, and all the Duhamel integrals in Maxwell's equations (6) replaced by products of the electric field and medium parameters.

3. Conduction current

Assuming zero initial value of the electric field in accordance with (A1), the conduction current is of the form:

$$\mathbf{j_c}(t) = \sigma * \mathbf{E} = \sigma(0)\mathbf{E}(t) + \int_0^t \frac{\partial \sigma(\lambda)}{\partial \lambda} \mathbf{E}(t-\lambda) d\lambda\,. \quad (7)$$

Therefore, there exists in a dispersive medium, the additional (compared to non-dispersive one) conduction current proportional to the time derivative of conductivity. If to assume the change of conductivity with time is slow enough to neglect the last term in (7), then the conduction current is described by the first term of (7) only and reacts the same as in non-dispersive medium.

It is also seen from (7) that $\mathbf{j_c}(0) = \sigma(0)\mathbf{E}(0) = 0$. We assume also that **E***(t)* tends with time to some stationary value **E***(∞).* One observes then from (7) that $\mathbf{j_c}(\infty) = \sigma(\infty)\mathrm{E}(\infty)$.

Suppose now that the electric field applied to the dispersive medium has the form of the step-function $\mathbf{E}(t) = \mathbf{E} \cdot 1(t)$. In this case the equation (7) gives

$$\mathbf{j_c}(t) = \sigma(t)\mathbf{E} \cdot 1(t), \quad (8)$$

from which it is clear that $\sigma(t)$ can make sense of the transient conductivity or the reaction of the dispersive medium to the step-change of the voltage applied (which is typical for the case of induction energizing the medium).

4. Displacement current

Assuming again zero initial value of the electric field in accordance with (A2), the displacement current is of the form:

$$\mathbf{j_d}(t)=\frac{\partial}{\partial t}(\varepsilon*\mathbf{E})=\varepsilon(0)\frac{\partial\mathbf{E}(t)}{\partial t}+\frac{\partial\varepsilon(t)}{\partial t}|_{t=0}\mathbf{E}(t)+\int_0^t\frac{\partial^2\varepsilon(\lambda)}{\partial\lambda^2}\mathbf{E}(t-\lambda)d\lambda. \qquad (9)$$

Therefore, exist in dispersive medium the additional (compared to non-dispersive one) displacement currents proportional to the first and second time derivatives of permittivity.

If the change of permittivity with time is so slow that it is justified to neglect the last term in (9), then the displacement current equals

$$\mathbf{j_d}(t)=\varepsilon(0)\frac{\partial\mathbf{E}(t)}{\partial t}+\frac{\partial\varepsilon(t)}{\partial t}|_{t=0}\mathbf{E}(t). \qquad (10)$$

The first term in (10) presents the common displacement current with constant value of permittivity $\varepsilon(0)$ like in non-dispersive medium. The second term in (10) can be rather treated as the additional conduction current caused by the change of permittivity with time. Then, instead of the first term of (7), one has to write the summary conduction current in dispersive medium with slow time change of electric parameters as

$$\mathbf{j}_{cs}=\gamma(0)\mathbf{E}(t), \qquad (11)$$

where the summary conductivity (admittivity)

$$\gamma(0)=\sigma(0)+\frac{\partial\varepsilon(t)}{\partial t}|_{t=0}. \qquad (12)$$

It is also seen from (10) that $\mathbf{j_d}(0)=\varepsilon(0)\frac{\partial\mathbf{E}(t)}{\partial t}|_{t=0}(0)$ and $\mathbf{j_d}(\infty)=\varepsilon(0)\frac{\partial\mathbf{E}(t)}{\partial t}|_{t\to\infty}+\frac{\partial\varepsilon(t)}{\partial t}|_{t\to\infty}\mathbf{E}(\infty)$.

In case of the step changed electric field or voltage is applied to the dispersive medium, the equation (9) gives

$$\mathbf{j_d}(t)=\varepsilon(0)\mathbf{E}\delta(\mathrm{t})+\frac{\partial\varepsilon(\mathrm{t})}{\partial t}\mathbf{E}\cdot 1(t),\ \text{or}\ =\frac{\partial\varepsilon(\mathrm{t})}{\partial t}\cdot\mathbf{E}\ \text{ at } t>0, \qquad (13)$$

from which it is clear that in this case (either in dispersive medium or not) the normal displacement current proportional to the constant permittivity $\varepsilon(0)$ does not exist at $t>0$.[2]

As for the dispersive medium, it makes sense to consider the time derivative of the transient permittivity $\frac{\partial\varepsilon(t)}{\partial t}$ as the reaction of the medium to the step-change of voltage applied. This reaction produces not the displacement current, rather the additional conduction one. Then, according to (8) and (13), the summary current (one

[2] The position is different in case of the step changed current is applied to the medium (which is typical for galvanic energising the medium and not considered here).

can say – the summary conduction current) at *t*>0 due to the step-changed voltage applied can be written as

$$\mathbf{j}_{\mathbf{cs}} = \left[\sigma(t) + \frac{\partial \varepsilon(t)}{\partial t} \right] \mathbf{E}, \qquad (14)$$

and the summary transient conductivity (admittivity) - as

$$\gamma(t) = \sigma(t) + \frac{\partial \varepsilon(t)}{\partial t}. \qquad (15)$$

Therefore, parameters σ and ε are not to be considered in dispersive media as independent ones. In particular, the time change of permittivity creates an additional conduction current.

5. Equations of the second order in time domain

By taking in system (6) *curl* from both sides of (I) and substituting (II, III) into (I) one obtains for the electric field:

$$\Delta \mathbf{E} = \mu \left[\frac{\partial}{\partial t} (\sigma * \mathbf{E}) + \frac{\partial^2}{\partial t^2} (\varepsilon * \mathbf{E}) \right], \qquad (16)$$

or, by using several times the auxiliary formulae (A1) and (A2),

$$\Delta \mathbf{E} = \mu \left[\begin{array}{l} \sigma(0) \frac{\partial \mathbf{E}(t)}{\partial t} + \frac{\partial \sigma(t)}{\partial t}\Big|_{t=0} \mathbf{E}(t) + \int_0^t \frac{\partial^2 \sigma(\lambda)}{\partial \lambda^2} \mathbf{E}(t-\lambda) d\lambda + \\ + \varepsilon(0) \frac{\partial^2 \mathbf{E}(t)}{\partial t^2} + \frac{\partial \varepsilon(t)}{\partial t}\Big|_{t=0} \frac{\partial \mathbf{E}(t)}{\partial t} + \frac{\partial^2 \varepsilon(t)}{\partial t^2}\Big|_{t=0} \mathbf{E}(t) + \int_0^t \frac{\partial^3 \varepsilon(\lambda)}{\partial \lambda^3} \mathbf{E}(t-\lambda) d\lambda \end{array} \right]. \qquad (17)$$

Equation (17) is presented by two groups of terms originally related to the conductivity and its change with time (line 1), as well as to permittivity and its change with time (line 2). If to assume the change of parameters *σ(t)* and *ε(t)* with time is slow enough to neglect last terms in both lines of (17), then

$$\Delta \mathbf{E} = \mu \left[\left(\sigma(0) + \frac{\partial \varepsilon(t)}{\partial t}\Big|_{t=0} \right) \frac{\partial \mathbf{E}(t)}{\partial t} + \left(\frac{\partial \sigma(t)}{\partial t}\Big|_{t=0} + \frac{\partial^2 \varepsilon(t)}{\partial t^2}\Big|_{t=0} \right) \mathbf{E}(t) + + \varepsilon(0) \frac{\partial^2 \mathbf{E}(t)}{\partial t^2} \right]. \quad 18)$$

If two fore-last terms in both lines of (17) are also abandoned, then

$$\Delta \mathbf{E} = \mu \left[\gamma(0) \frac{\partial \mathbf{E}(t)}{\partial t} + \varepsilon(0) \frac{\partial^2 \mathbf{E}(t)}{\partial t^2} \right], \qquad (19)$$

where $\gamma(0)$ is defined by (12).

It is possible in the last case to speak about the quasi-stationary EM field in dispersive medium, where normal displacement current can be also abandoned. Then

$$\Delta\mathbf{E} = \mu\gamma(0)\frac{\partial\mathbf{E}(t)}{\partial t}. \qquad (20)$$

Equations (19 and 20) look similar to common Telegrapher's and heat-flow equations respectively, except for the difference between admittivity and common conductivity *σ=const.* That means, the replacing of both time-independent electric parameters by very slow changeable once only is possible in time domain. Otherwise the basic equations of the second order are different.

Similarly, by taking into account in addition A3, for the magnetic field:

$$\Delta\mathbf{H} = \mu\left[\left(\sigma * \frac{\partial\mathbf{H}}{\partial t}\right) + \frac{\partial}{\partial t}\left(\varepsilon * \frac{\partial\mathbf{H}}{\partial t}\right)\right] = \mu\left[\frac{\partial}{\partial t}(\sigma * \mathbf{H}) + \frac{\partial^2}{\partial t^2}(\varepsilon * \mathbf{H})\right] =$$

$$= \mu\left[\begin{array}{l}\sigma(0)\dfrac{\partial\mathbf{H}(t)}{\partial t} + \dfrac{\partial\sigma(t)}{\partial t}\Big|_{t=0}\mathbf{H}(\mathbf{t}) + \displaystyle\int_0^t \dfrac{\partial^2\sigma(\lambda)}{\partial\lambda^2}\mathbf{H}(t-\lambda)d\lambda + \\ + \varepsilon(0)\dfrac{\partial^2\mathbf{H}(t)}{\partial t^2} + \dfrac{\partial\varepsilon(t)}{\partial t}\Big|_{t=0}\dfrac{\partial\mathbf{H}(t)}{\partial t} + \dfrac{\partial^2\varepsilon(t)}{\partial t^2}\Big|_{t=0}\mathbf{H}(t) + \displaystyle\int_0^t \dfrac{\partial^3\varepsilon(\lambda)}{\partial\lambda^3}\mathbf{H}(t-\lambda)d\lambda\end{array}\right]. \qquad (21)$$

Therefore, all equations (17-20) above for the electric field are same for the magnetic one.

6. Equationsof the EM field in frequency domain

In accordance with the convolution theorem

$$f(t) * \mathbf{F}(t) = L^{-1}\left[\frac{f(p)}{p}\cdot\mathbf{F}(p)\right], \text{ or } \dot{f}(t) * \mathbf{F}(t) = L^{-1}\left[f(p)\cdot\mathbf{F}(p)\right], \qquad (22)$$

provided $f(t) = L^{-1}[f(p)/p]$, $\dot{f}(t) = L^{-1}[f(p)]$ and $\mathbf{F}(t) = L^{-1}[\mathbf{F}(p)]$, where L^{-1} is the symbol of the inverse Fourier/Laplace transform with *p=-iω*.

The subject to study in time domain is normally the parameter *f(t)*, which is the transient characteristic of the medium or reaction of the medium to the step-type energizing the EM field described by the 1(*t*) function with the spectrum 1/*p*, whereas $\dot{f}(t)$ is the impulse characteristic of the medium or reaction of the medium to the pulse-type energizing the EM field described by the *δ*(*t*) function with the uniform spectrum 1(*p*). The subject to study in frequency domain is the frequency characteristic *f(p)* of the medium, corresponding to the uniform spectrum. That is way

one has to consider the Duhamel integral *f*(*t*)**F*(*t*) in time domain and the product *f*(*p*)*F*(*p*) in frequency domain.
Then, the system (6) of Maxwell's equations looks in frequency domain like

$$\begin{aligned}&\mathrm{I.}\, curl\mathbf{E}(p) = p\mathbf{B}(p), \mathrm{II.}\, curl\mathbf{H}(p) = \sigma(p)\mathbf{E}(p) + p\varepsilon(p)\mathbf{E}(\mathbf{p}),\\ &\mathrm{III.}\, div\mathbf{E}(p) = 0, \mathrm{IV.}\, div\mathbf{B}(\mathrm{p}) = 0\end{aligned} \quad . \qquad (23)$$

Corresponding transforms of equations (16) and (21) of the second order are of the form

$$\Delta(\mathbf{E},\mathbf{H}) = \mu\{p\sigma(p)[\mathbf{E}(\mathbf{p}),\mathbf{H}(\mathbf{p})] + p^2\varepsilon(p)[\mathbf{E}(\mathbf{p}),\mathbf{H}(\mathbf{p})]\}, \qquad (24)$$

which brings us with $p=-i\omega$ to the Helmholtz equation

$$\Delta(\mathbf{E},\mathbf{H}) + k^2(\mathbf{E},\mathbf{H}) = 0 \qquad (25)$$

with the square wave-number

$$k^2 = i\omega\mu\sigma(\omega) + \omega^2\mu\varepsilon(\omega) \qquad (26)$$

and frequency dependent conductivity $\sigma(\omega)$ and permittivity $\varepsilon(\omega)$.
Therefore, the EM field in frequency domain (contrary to time domain) is described by one and the same equation of the second order, both in non-dispersive and dispersive media. That means the spectral method is a universal one.
It should be noted that while studying IP effects in frequency domain (similar to time domain) the separate analyses of dispersion of parameters σ or ε is not correct, including the quasi-stationary case. The combination of both parameters known as complex admittivity $\gamma(\omega) = \sigma(\omega) - i\omega\varepsilon(\omega)$ is to be analyzed. Otherwise, the interconnection between dispersions of conductivity and permittivity, clearly seen from time domain formulae, can not be detected by spectral method.

8. Conclusions

1. The system of Maxwell's equations themselves written in canonical form is a universal means of describing EM field both in non-dispersive and dispersive media, provided material correlations are presented by Duhamel integrals. Terms such as "non-classical electromagnetics/geoelectrics", as applied to IP effects in geo-electromagnetics, make no sense.
2. In time domain, additional conductivity and displacement currents appear due to changes in conductivity and permittivity parameters with time.
3. Equations of the second order in dispersive medium are in time domain different from Telegrapher's and heat-flow equations. Even, in case of a very slow change in the parameters over time, only the equation format is the same, but they still include the summary admittivity consisting of normal conductivity and time derivative of permittivity. Therefore, methods of replacing constant parameters with time variable ones are in time domain principally not correct and are possible in case of a very slow time change of both electric parameters only..

4. The equation of the second order in frequency domain is in dispersive medium the same Helmholtz equation. The method of substituting constant parameters by frequency variable ones is in frequency domain correct. The complex admittivity including both frequency dependent conductivity and permittivity is to be analyzed, inclusive the quasi-stationary case. Otherwise, the interconnection between dispersions of conductivity and permittivity, clearly seen from time domain formulae, can not be detected by spectral method.

Acknowledgments

The author would like to express his thanks to Prof. P. Weidelt (Germany) and Dr. A. Petrov (Russia) for many stimulating discussions.

Appendix A: Auxiliary formulae

1. By taking account zero initial values of the EM field component one obtains:

$$f*\mathbf{F}=\frac{d}{dt}\int_0^t f(\lambda)\cdot\mathbf{F}(t-\lambda)d\lambda= f(t)\mathbf{F}(0)+\int_0^t f(\lambda)\frac{\partial}{\partial\lambda}\mathbf{F}(t-\lambda)d\lambda=-\int_0^t f(\lambda)\frac{\partial\mathbf{F}(t-\lambda)}{\partial(t-\lambda)}d(t-\lambda)=-\int_0^t f(\lambda)d\mathbf{F}(t-\lambda)=$$

$$=-\int_0^t\{\ d[f(\lambda)\mathbf{F}(t-\lambda)]-\mathbf{F}(t-\lambda)df(\lambda)\ \}=-f(\lambda)\mathbf{F}(t-\lambda)\Big|_0^t+\int_0^t\mathbf{F}(t-\lambda)df(\lambda)=-[f(t)\mathbf{F}(0)-f(0)\mathbf{F}(t)]+\int_0^t\mathbf{F}(t-\lambda)df(\lambda)=$$

$$= f(0)\mathbf{F}(t)+\int_0^t\frac{\partial f(\lambda)}{\partial\lambda}\mathbf{F}(t-\lambda)d\lambda.$$

(A1)

2.

$$\frac{\partial}{\partial t}(f*\mathbf{F})=\frac{d}{dt}\left[\frac{d}{dt}\int_0^t f(\lambda)\mathbf{F}(t-\lambda)d\lambda\right]=\text{...see A1...}=\frac{d}{dt}\left[f(0)\mathbf{F}(t)+\int_0^t\mathbf{F}(t-\lambda)\frac{\partial f(\lambda)}{\partial\lambda}d\lambda\right]=$$

$$= f(0)\frac{d\mathbf{F}(t)}{\partial t}+\frac{\partial f}{\partial t}*\mathbf{F}= f(0)\frac{\partial\mathbf{F}(t)}{\partial t}+\frac{\partial f(t)}{\partial t}\Big|_{t=0}\mathbf{F}(t)+\int_0^t\frac{\partial^2 f(\lambda)}{\partial\lambda^2}\mathbf{F}(t-\lambda)d\lambda.$$

(A2)

3. If to reverse roles of functions *f* and F in A2, then

$$\frac{\partial}{\partial t}(\mathbf{F}*f)=\mathbf{F}(0)\frac{\partial f(t)}{\partial t}+\frac{\partial\mathbf{F}}{\partial t}*f = f*\frac{\partial\mathbf{F}}{\partial t}, \qquad \text{(A3)}$$

Appendix B: Dispersion of magnetic permeability

Suppose now that the magnetic permeability is variable parameter, whereas the conductivity and permittivity are constant. Then the material correlations are:

$$\mathbf{j_d}=\sigma\mathbf{E},\quad \mathbf{D}=\varepsilon\mathbf{E},\quad \mathbf{B}=\mu*\mathbf{H}\ ,$$

and Maxwell's equations:

$$\text{I.}\,curl\mathbf{E} = -\frac{\partial(\mu * \mathbf{H})}{\partial t},\ \text{II.}\,curl\mathbf{H} = \sigma\mathbf{E} + \varepsilon\frac{\partial \mathbf{E}}{\partial t},\ \text{III}\,div\mathbf{E} = 0,\ \text{IV}\,div\mathbf{B} = 0.$$

By transformations (similar to those used in section 5 of the main text) it can be shown that equation of the second order in time domain for the electric field (and similarly for the magnetic field) is of the form:

$$\Delta\mathbf{E} = \mu_v\left[\sigma\frac{\partial}{\partial t}(\overline{\mu} * \mathbf{E}) + \varepsilon\frac{\partial^2}{\partial t^2}(\overline{\mu} * \mathbf{E})\right],$$ or

$$\Delta\mathbf{E} = \mu_v\left[\begin{array}{l}\sigma\left\langle\ \overline{\mu}(0)\frac{\partial\mathbf{E}(t)}{\partial t} + \frac{\partial\overline{\mu}(t)}{\partial t}\Big|_{t=0}\mathbf{E}(t) + \int_0^t \frac{\partial^2\overline{\mu}(\lambda)}{\partial\lambda^2}\mathbf{E}(t-\lambda)d\lambda\ \right\rangle + \\ + \varepsilon\left\langle\ \overline{\mu}(0)\frac{\partial^2\mathbf{E}(t)}{\partial t^2} + \frac{\partial\overline{\mu}(t)}{\partial t}\Big|_{t=0}(0)\frac{\partial\mathbf{E}(t)}{\partial t} + \frac{\partial^2\overline{\mu}(t)}{\partial t^2}\Big|_{t=0}\mathbf{E}(t) + \int_0^t \frac{\partial^3\overline{\mu}(\lambda)}{\partial\lambda^3}\mathbf{E}(t-\lambda)d\lambda\ \right\rangle\end{array}\right]$$

with $\overline{\mu}(t) = \mu(t)/\mu_v = 1 + 4\pi\chi(t),\ \ \mu_v = 4\pi\cdot 10^{-7}$ H/m .

In case of slow change of magnetic permeability, where last two terms in both lines of the previous equation are abandoned:

$$\Delta\mathbf{E} = \mu_v\left[\gamma_\mu(0)\frac{\partial\mathbf{E}(t)}{\partial t} + \varepsilon\overline{\mu}(0)\frac{\partial^2\mathbf{E}(t)}{\partial t^2}\right],$$

where the summary admittivity

$$\gamma_\mu(0) = \sigma\overline{\mu}(0) + \varepsilon\frac{\partial\overline{\mu}(t)}{\partial t}\Big|_{t=0}.$$

The last term of summary admittivity presents the additional conductivity due to the change of the magnetic permeability over time.

Therefore, except for small necessary changes, equations of the EM field in the media with dispersive magnetic permeability are in time domain the same as for the media with dispersive electric parameters. It is evident also that in frequency domain the equation of the second order is the same Helmholtz equation with the squared wave number containing the frequency dependent magnetic permeability $\mu(\omega)$.

Referenses

Kamenetsky, F. M., 2002. Equations of the electromagnetuc field in dispersive media. *The 16ts Workshop on Electromagnetic induction in the Earth.* June 16-22 2002, The College of Santa Fe, Santa Fe, New Mexico, USA.

Kamenetsky, F. M., Novikov, P.V., 1997. A physical study of low-frequency dispersion of rock conductivity in time-domain electromagnetics, *Geophysical Prospecting*, 3, 421-434.

Kormiltsev, V. V., Mesentsev, A. N., 1989. Electric prospecting of polarising media, *Ural division of Ac. Sc. of the USSR*, Sverdlovsk, 127 pp.

Porstendorfer, G., 1987, Perspectives to extract information on electrical conductivity, induced polarization and magnetic susceptibility of geological media from variation of the natural electromagnetic field of the Earth, *HHI-Report*, 21, Berlin, 115-121.

Safonov, A. S., Mushin, I. A., Kiselev, E. S., Goryunov, A. S., 1996, Structural-formation model as a physics-geological basement of high-resolving geo-electromagnetics. Geophysika, 2, 12-23.

Scheinmann, S. M., 1969. Modern theoretical basement of the electric prospecting, Nedra, Moscow, 224 pp.

Svetov, B. S., 1996. Non-classical geoelectrics, *Physics of the solid Earth*, v. 31, 8, 639-647.

Wait, J. R., 1959. The variable Frequency method. Overvoltage research and geophysical application, L., N. Y., Paris, Los Angeles, 1325-1330.